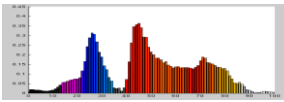


Dr. Qadri Hamarsheh

Signals and Systems

Lecture 5

Linear Time-Invariant Systems
(LTI Systems)

Outline

- **Introduction.**
- **Mathematical Models Types (Representations).**
- **Examples:**
  - ✓ **Continuous-time systems: RC Circuit.**
  - ✓ **Discrete-time systems: Moving Average Filter.**

Introduction

We can define the system as a mathematical model that represents the **transformation** of some input signal $x(t)$ or $x[n]$ into an output signal $y(t)$ or $y[n]$, the graphical representation of this idea is shown in **figure 3-1**.

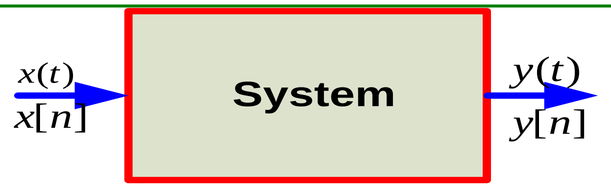


Figure 3-1 Systems Structure

A real physical implementation of this structure is the relationship between the current and voltage of resistor as shown in **figure 3-2**.

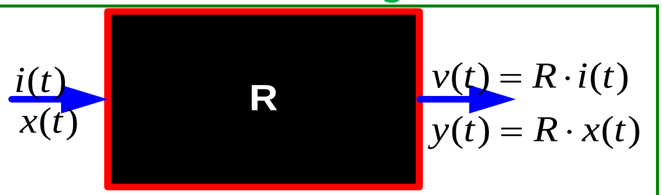
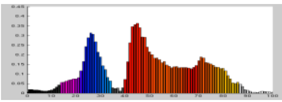


Figure 3-2

System can be thought of as a transformation or operator that we will denote by $\hat{T}$ (see **figure 3-3**), this operator maps an input sequence to an output sequence:

$$y(t) = \hat{T}\{x(t)\}.$$

An example of such operator is a **derivative operator** $\hat{T} = \frac{d}{dt}$



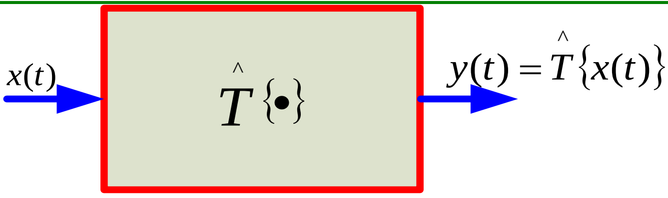


Figure 3-3

Mathematical Models Types (Representations)

1. **Input-Output Representations (Models)** that describe the relationship between the input and output signals of a system, some of these representations are the following:
   - 1.1 **Convolution Model**.
   - 1.2 **Input-Output Difference or Differential Equations**.
   - 1.3 **Fourier Transform** Representation (special case of Transfer Function Representation).
   - 1.4 **Transfer Function** Representation.
2. **State or Internal Representation (Model)** that describes the relationship among the **input**, **state** and the **output** signals of a system.

The first two representations of input-output representations and the state representation are **time-domain models** (function of time).The others are **frequency-domain models** (function of frequency), both time-domain and frequency-domain models are used in system analysis and design.

Examples

1. **Continuous-time systems: RC Circuit.**
2. **Discrete-time systems: Moving Average Filter.**

Continuous-time systems (RC Circuit):

Using input-output model, we can describe the RC Circuit, in this case, the input signal, $x(t)$ is the current $i(t)$ into the parallel connection of RC circuit and the output signal, $y(t)$ is the voltage across the capacitor $V_C(t)$ (see **figure 3-4**).

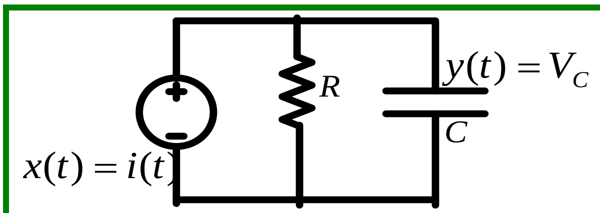


Figure 3-4 RC Circuit

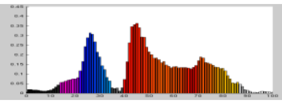
By **Kirchhoff's** current low:

$$i_C(t) + i_R(t) = i(t)$$

where $i_C(t)$- is the current in the capacitor, $i_R(t)$-is the current in the resistor.
From physics:

$$i_C(t) = c \cdot \frac{dV_C(t)}{dt} = c \cdot \frac{dy(t)}{dt}$$

and



$$i_R(t) = \frac{1}{R} \cdot V_C(t) = \frac{1}{R} \cdot y(t)$$

then :

$$c \cdot \frac{dy(t)}{dt} + \frac{1}{R} y(t) = i(t) = x(t)$$

This is the input-output differential equation of the circuit.
The output, $y(t)$, resulting from an input, $x(t)$, can be generated by solving the input-output differential equation:
For example: suppose that $x(t) = u(t)$ - unit step function and the initial condition $y(0) = 0$, then the differential equation of the RC circuit will be in the form

$$c \cdot \frac{dy(t)}{dt} + \frac{1}{R} y(t) = 1, \quad t > 0$$

Using **Matlab** or using **Laplace Transform**, the solution is equal to

$$y(t) = R\left[1 - e^{-(\frac{1}{RC}) \cdot t}\right], \quad t \geq 0$$

The output is called the step response

Discrete-time systems (Moving Average Filter):

The N −point Moving Average (**MA**) Filter is a discrete-time system given by the input-output relationship:

$$y[n] = \frac{1}{N}[x[n] + x[n-1] + x[n-2] + ... + x[n-N+1]]$$

where N −is any positive integer number, $x[n]$ −the filter' input signal and $y[n]$ −the filter' resulting output response (see **figure 3-5**).

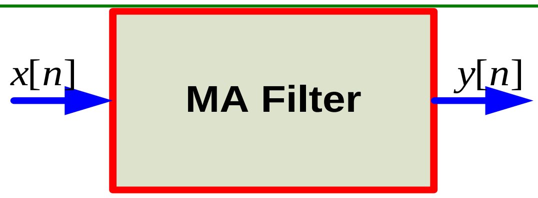


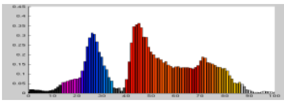
Figure 3-5 Moving Average Filter

For example, the **3-point MA** filter is given by:

$$y[n] = \frac{1}{3}[x[n] + x[n-1] + x[n-2]]$$

- ☒ The output $y[n]$ at time n of the N −point MA filter is the average of the N input values $x[n], x[n-1], x[n-2], ... x[n-N+1]$.
- ☒ The filter is referred to as "**Moving Average Filter**" or "**Running Average Filter**" because we compute the next value $y[n+1]$ of the output moving the range of time points over which the filter output is computed. In particular,

$$y[n+1] = \frac{1}{N}[x[n+1] + x[n] + x[n-1] + ... + x[n-N+2]]$$



- ☒ MA filters are often used to reduce the magnitude of the noise that may present in a signal.

Suppose that the input signal $x[n]$ is given as shown in **figure 3-6** and expressed by the formula:

$$x[n] = s[n] + e[n]$$

Where $s[n]$ is the smooth part of $x[n]$ and $e[n]$ is the erratic or noisy part of $x[n]$, then the output $y[n]$ of the $N-$point MA filter is given by:

$$y[n] = \frac{1}{N}\left[s[n] + s[n-1] + s[n-2] + \ldots + s[n-N+1]\right] +$$
$$\frac{1}{N}\left[e[n] + e[n-1] + e[n-2] + \ldots + e[n-N+1]\right]$$

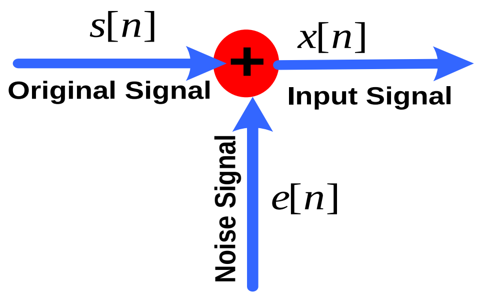


Figure 3-6

The noisy part of the MA filter output $y[n] \Rightarrow$ is the average of the noise values $e[n] + e[n-1] + e[n-2] + \ldots + e[n-N+1]$ which is equal to

$$\frac{1}{N}\left[e[n] + e[n-1] + e[n-2] + \ldots + e[n-N+1]\right]$$

In this equation, if $e[n]$ varies randomly about zero, the noisy term can be made as small as desired (in theory) by taking N to be sufficiently large.

- ☒ We can simulate the work of MA filter using Matlab program to filter some input data.

```
>> x = [3 4 5 6 1 3 2 1 8 9 13 10];
>> MA = (1/12)*sum(x)
Solution:
MA = 5.4167
```